

# Non-Empty Quantum Dot as a Spin-Entangler

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## Abstract

We consider a three-port single-level quantum dot system with one input and two output leads. In stead of considering an empty dot, we study the situations that two input electrons co-tunnel through the quantum dot occupied by one or two dot electrons. We show that electron entanglement can be generated via the co-tunneling processes when the dot is occupied by two electrons, yielding non-local spin-singlet states at the output leads. When the dot is occupied by a single electron, we show that by carefully selecting model parameters non-local spin-triplet electrons can also be obtained at the output leads if the final dot electron has the same spin as that of the initial dot electron.

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## I. INTRODUCTION

The quantum theory of computation and information has been rapidly developed in the past years on the basis of quantum mechanics. The quantum computation and communication require quantum operations (quantum gates) on quantum bits (qubits). Unlike the classical bit of information that always takes either "0" or "1" value, the qubit is a two-level quantum system which can be prepared in arbitrary superpositions of its two states, usually denoted as  $|0\rangle$  and  $|1\rangle$ . The principle of superposition also applies to input and output registers of quantum circuits. The principle of superposition has no correspondence in classical information theory and leads to the so-called quantum parallelism [1] and other novel properties in quantum computation theory [2]. Up to date, several quantum systems have been considered as qubits and quantum gates. For example, nuclear magnetic resonance quantum computing [3], neutral atoms [4], low-capacitance Josephson junctions [5], trapped ions [6], cavity QED schemes [7], and the spin states of coupled semiconductor quantum dots [8] etc., are proposed or have been used to implement qubits and universal quantum gates for quantum computation.

In addition to the principle of superposition, the nonlocal nature of entangled quantum states also play important roles in quantum computation and communication. The Einstein-Podolsky-Rosen (EPR) pairs [9] are essential in many striking phenomena such as quantum teleportation [10], dense coding [11], secure quantum communication [12], and the tests of the violation of Bell's inequality [13]. These interesting phenomena have been tested in experimental demonstrations which used entangled photons as EPR pairs. Although such tests exist for photons, they have not been tested for electrons since it is difficult to produce and detect entangled electrons in solid state environment. Recently, several proposals have been raised to generate [14, 15, 16] and detect [17] entangled electrons. There are several main motivations for these proposals. First, electron spins in semiconductor environment have been demonstrated to have long dephasing times approaching microseconds [18]. Since the qubits defined as electron spins are mobile, they can be good candidates for implementing quantum communication in solid state environment. Second, the EPR electrons can be spatially separated from each other thus the control of nonlocality can be implemented. Third, solid-state qubits may be integrated into large quantum networks which is required for the realization of quantum computers.

Among the proposals for generation of electron entanglement, many of them propose quantum dots as the spin-entanglers [14, 15]. Recher *et al.* [15] suggest using two coupled quantum dots that are coupled to one s-wave superconductor input lead and two normal Fermi output leads as a electron spin-entangler. Two electrons from the superconductor tunnel coherently into different output leads via different quantum dots by the Andreev process in the Coulomb blockade regime with the presence of voltage bias. Oliver *et al.* [14] suggest using an empty single-level quantum dot with one input and two output leads as the spin-entangler. The leads are nondegenerate and of narrow widths in energy so that they act as energy filters for the input and output electrons. The leads are arranged to suppress any single-electron tunneling process. Only two input electrons can co-tunnel from the input lead into different output leads via the dot. In this scheme, the double occupancy of the dot will incur an on-site Coulomb interaction which then mediates electron entanglement. Therefore, two input electrons with opposite spins will be in the spin-singlet state at the output leads after co-tunneling through the dot.

In this paper, we showed that the quantum dot needs not to be an empty one to become a spin-entangler as in the scheme proposed by Oliver *et al.*[14]. In contrast, the quantum dot can be either occupied by a single electron or by two electrons with different spins. When there is one electron occupying the dot, a net spin-triplet amplitude can be obtained at the output by co-tunneling two electrons if the dot electron does not flip its spin after the co-tunneling processes. When there are two electrons with different spins occupying the dot, a net spin-singlet state is obtained after co-tunneling two input electrons. The other final states such as the spin-triplet and the same-spin states at the output leads destructively interfere. It is noted that the on-site Coulomb interaction still plays the role of entanglement mediator as usual. Therefore when the Coulomb interaction is turned off, all possible final states destructively interfere thus give vanishing transition amplitudes.

## II. THE QUANTUM DOT SYSTEM

The arrangement of the quantum dot system and its energy band diagram are given in Ref.[14] and also in Fig.1. Through out this paper we assume that the quantum dot has only a single spin-degenerate energy level, thus no single electron excitation inside the dot is allowed. According to Pauli's principle of exclusion, the dot can accommodate at most two

electrons with different spins. Therefore, same-spin electrons will not co-exist within the dot. An on-site Coulomb interaction is assumed to incur when two different-spin electrons occupy the dot. The quantum dot system is described by the Anderson Hamiltonian as follows:

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{V}, \\ \hat{H}_0 &= \sum_{\eta,k,\sigma} \varepsilon_{\eta,k} \hat{a}_{\eta,k,\sigma}^\dagger \hat{a}_{\eta,k,\sigma} + \sum_{\sigma} \varepsilon_d \hat{c}_{\sigma}^\dagger \hat{c}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}, \\ \hat{V} &= \sum_{\eta,k,\sigma} (V_{\eta} \hat{a}_{\eta,k,\sigma}^\dagger \hat{c}_{\sigma} + h.c.),\end{aligned}\tag{1}$$

where  $\eta \in (L, R_1, R_2)$  denotes the lead label,  $k$  is the lead electron momentum,  $\sigma$  is the electron spin,  $\hat{a}_{\eta,k,\sigma}^\dagger (\hat{a}_{\eta,k,\sigma})$  is the creation (annihilation) operator for the lead electrons,  $\hat{c}_{\sigma}^\dagger (\hat{c}_{\sigma})$  is the creation (annihilation) operator for the dot electrons,  $\hat{n}_{\sigma}$  is the number operator for the dot electrons with spin  $\sigma$ , and  $U$  denotes the charging energy. The tunneling matrix element between the dot and the lead states is given by  $V_{\eta}$ .

As shown in Fig.1(a), two electrons co-tunnel through the dot and become spatially separated at the output leads  $R_1$  and  $R_2$ . Any single-electron tunneling is forbidden in this model. This can be achieved by biasing the dot system such that no energy of a single left lead electron equals the energy of a right lead electron, i.e.,  $\varepsilon_{L,k} \neq \varepsilon_{R_1,k_1} \neq \varepsilon_{R_2,k_2}$ , and that the conservation of energy holds for virtual two-electron co-tunneling,  $\varepsilon_{L,k} + \varepsilon_{L,k'} = \varepsilon_{R_1,k_1} + \varepsilon_{R_2,k_2}$ . Through out the paper, we assign the dot level energy  $\varepsilon_d$  to be zero without loss of generality. The energy separation for the left lead electrons is required to be smaller than the energy separation for the right leads, i.e.,  $\Delta_L < \Delta_R$ . On the other hand, the widths of the lead energy bands are required to be narrow so that the leads act as energy filters. The energies of the two right leads are required to be below their quasi-Fermi levels such that the right leads are empty. The left lead is assumed to be filled of electrons. As we will show later, a net amplitude for the singlet-spin state at the output leads is filtered out from a two-electron input in the case that the dot is occupied by two electrons. However, several final states may have nonzero transition amplitudes when the dot is occupied by a single electron. In this case if the dot electron does not flip its spin after co-tunneling, a net amplitude for the triplet-spin state at the output leads can be obtained by carefully arranging the model parameters.

From the Hamiltonian given in Eq.(1), the lowest order contribution to the co-tunneling

processes is  $\mathcal{O}(V^4)$ . We calculate the transition amplitudes for two-electron co-tunneling processes by using the transition matrix formalism,

$$\hat{T}(\varepsilon_i) = \hat{V} \frac{1}{\varepsilon_i - \hat{H}_0} \hat{V} \frac{1}{\varepsilon_i - \hat{H}_0} \hat{V} \frac{1}{\varepsilon_i - \hat{H}_0} \hat{V}, \quad (2)$$

where  $\varepsilon_i$  is the energy of the initial state of the system and  $\hat{T}(\varepsilon_i)$  is the transition operator that is relevant to the two-electron co-tunneling. Thus the transition amplitude between the initial state  $|\varphi_i\rangle$  and the final state  $|\varphi_f\rangle$  is given by  $\langle \varphi_f | \hat{T}(\varepsilon_i) | \varphi_i \rangle$ .

### III. QUANTUM DOT OCCUPIED BY TWO ELECTRONS

In this section, we assume that the quantum dot is occupied by two electrons with different spins, i.e., spin up ( $\uparrow$ ) and spin down ( $\downarrow$ ). In general, the spins of the input electrons will be one of the following four combinations:  $(\sigma\sigma') = (\uparrow\downarrow), (\downarrow\uparrow), (\uparrow\uparrow)$  and  $(\downarrow\downarrow)$ . The initial four-electron state is given by

$$|\varphi_i\rangle = \hat{c}_{(\downarrow)}^+ \hat{c}_{(\uparrow)}^+ \hat{a}_{L,k,\sigma}^+ \hat{a}_{L,k',\sigma'}^+ |0\rangle, \quad (3)$$

where  $|0\rangle$  denotes the zero-particle state of the model system. The energies for the input electrons are denoted as  $\varepsilon_{L,k} = E_L + \Delta_L$  and  $\varepsilon_{L,k'} = E_L - \Delta_L$  such that the energy of the initial state is  $\varepsilon_i = 2E_L + U$ . Therefore, the co-tunneling amplitude for  $(\sigma\sigma') = (\uparrow\downarrow)$  can be obtained from the amplitude for  $(\sigma\sigma') = (\downarrow\uparrow)$  by replacing  $\Delta_L$  with  $-\Delta_L$  in the amplitude. On the other hand, since the tunneling matrix element  $V_\eta$  is spin-blind the transition amplitude for the case  $(\sigma\sigma') = (\uparrow\uparrow)$  should be equal to the amplitude of the case  $(\sigma\sigma') = (\downarrow\downarrow)$ . This means, only two co-tunneling processes are needed to be calculated.

We consider first the case  $(\sigma\sigma') = (\downarrow\uparrow)$ . In this case, we cannot have two same-spin electrons at the output leads. Since if there exist two same-spin electrons at the output leads, then two same-spin electrons must co-exist within the quantum dot, thus violate Pauli's exclusive principle. Therefore, only electrons of different spins can be obtained at the output leads. From Eq.(2), the time-ordering operator leads to twelve virtual paths by which the input electrons can co-tunnel through the dot into output leads. Six of the twelve paths that have "direct" time ordering are shown in Fig.2. The remaining six virtual paths that have "exchange" time ordering are obtained simply by exchanging the output leads  $R_1$  and  $R_2$ . In general, the portion of the transition amplitude from the "exchange paths"

acquires a relative minus sign and can be easily obtained by simply replacing  $\Delta_R$  with  $-\Delta_R$  in the amplitude from the "direct paths". After all, the transition amplitudes for the singlet  $|s\rangle$  and triplet  $|t\rangle$  final states are

$$\begin{aligned} |s\rangle, |t\rangle &\equiv \frac{1}{\sqrt{2}} \hat{c}_{(\uparrow)}^\dagger \hat{c}_{(\downarrow)}^\dagger (\hat{a}_{R_1(\uparrow)}^\dagger \hat{a}_{R_2(\downarrow)}^\dagger \mp \hat{a}_{R_1(\downarrow)}^\dagger \hat{a}_{R_2(\uparrow)}^\dagger) |0\rangle \\ \langle t | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= 0 \\ \langle s | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= \frac{2\sqrt{2}U(E_L - U)V_L^{*2}V_{R_1}V_{R_2}}{(2E_L - U)((E_L - U)^2 - \Delta_L^2)((E_L - U)^2 - \Delta_R^2)}. \end{aligned} \quad (4)$$

The triplet transition amplitude destructively interfere, similar to the case of co-tunneling through an empty dot [14]. The singlet transition amplitude is also destructive, but not complete when the on-site Coulomb interaction  $U \neq 0$ . However, as different to the case of co-tunneling an empty quantum dot, all virtual paths in Fig.2 are  $U$ -dependent thus are suppressed as  $U \rightarrow \infty$ .

The remaining cases to be considered are  $(\sigma\sigma') = (\uparrow\uparrow)$  and  $(\downarrow\downarrow)$ . In these cases, the output electrons at the two right leads always have the same spins as those of the input electrons. Take the case  $(\sigma\sigma') = (\uparrow\uparrow)$  as an example, the final state of the dot system must be  $\hat{c}_{(\downarrow)}^\dagger \hat{c}_{(\uparrow)}^\dagger \hat{a}_{R_1(\uparrow)}^\dagger \hat{a}_{R_2(\uparrow)}^\dagger |0\rangle$ . The time-ordering operator in Eq.(2) thus leads to four unique time orderings. We find that these four virtual paths destructively interfere and give vanishing amplitude, independent of the on-site Coulomb energy  $U$ .

Base on the above calculation, we now conclude that only net singlet transition amplitude can be obtained in the case of two electrons occupying the quantum dot. The left lead can be a Fermi metal which is filled of electrons such that the input electrons can be an arbitrary selection of two electrons of various types of spins. Any non-vanishing co-tunneling of two electrons from the left lead thus generates entangled spin-singlet electrons at the output leads.

#### IV. QUANTUM DOT OCCUPIED BY A SINGLE ELECTRON

In this section, we assume that the quantum dot is occupied by a single electron with definite spin  $\sigma$ . Without loss of generality, we assume the spin of the dot electron to be spin-down  $\sigma = (\downarrow)$ . Again, there are four possible combinations for the spins of the input electrons,  $(\sigma\sigma') = (\uparrow\downarrow), (\downarrow\uparrow), (\uparrow\uparrow)$  and  $(\downarrow\downarrow)$ . Again, the first two combinations are actually the same under the transformation  $\Delta_L \rightarrow -\Delta_L$  except for an extra factor  $(-1)$  which

is required due to the exchange of operators for input electrons. The last two cases are same-spin electrons co-tunneling through the dot. However, these two cases are intrinsically different from each other since one of them must have different spin as that of the dot electron.

We consider first the cases of same-spin co-tunneling. For the case of  $(\sigma\sigma') = (\uparrow\uparrow)$ , the initial state of the quantum dot system is  $\hat{c}_{(\downarrow)}^+ \hat{a}_{L,k,(\uparrow)}^+ \hat{a}_{L,k',(\uparrow)}^+ |0\rangle$ . The output electrons at the two right leads may form a spin-triplet, a spin-singlet, or a same-spin ( $\uparrow\uparrow$ ) state which corresponds to a spin-up, a spin-up, or a spin-down dot electron confined within the dot, respectively. Therefore, the possible final states of the dot system are  $|\uparrow, s\rangle, |\uparrow, t\rangle \equiv \frac{1}{\sqrt{2}} \hat{c}_{(\uparrow)}^+ (\hat{a}_{R_1(\uparrow)}^+ \hat{a}_{R_2(\downarrow)}^+ \mp \hat{a}_{R_1(\downarrow)}^+ \hat{a}_{R_2(\uparrow)}^+) |0\rangle$ , and  $|\downarrow, \uparrow\uparrow\rangle \equiv \hat{c}_{(\downarrow)}^+ \hat{a}_{R_1(\uparrow)}^+ \hat{a}_{R_2(\uparrow)}^+ |0\rangle$ . For the case that a singlet or a triplet state is generated at the output leads, there are sixteen virtual paths by which the input electrons can virtually co-tunnel through the dot to the right leads. Among these virtual paths, eight out of the sixteen paths that correspond to "direct" time ordering are shown in Fig.3. The remaining eight paths that have "exchange" time ordering are due to the exchange of the output leads  $R_1$  and  $R_2$ . On the other hand, there are four unique virtual co-tunneling paths that contribute to the transition amplitude of a same-spin final state  $|\downarrow, \uparrow\uparrow\rangle$  at the output leads. Two of them can be viewed as having "direct" time ordering, and the remaining two virtual paths correspond to "exchange" time ordering. For the case of  $(\sigma\sigma') = (\downarrow\downarrow)$ , the initial state of the dot system is  $\hat{c}_{(\downarrow)}^+ \hat{a}_{L,k,(\downarrow)}^+ \hat{a}_{L,k',(\downarrow)}^+ |0\rangle$ , i.e., all input and dot electrons have the same spin. Therefore there is only one possible final state for the dot system,  $|\downarrow, \downarrow\downarrow\rangle \equiv \hat{c}_{(\downarrow)}^+ \hat{a}_{R_1(\downarrow)}^+ \hat{a}_{R_2(\downarrow)}^+ |0\rangle$ . Again, there are four paths that electrons can virtually co-tunnel through the quantum dot to the output leads. Two of the paths can be viewed as having "direct" time orderings, and the other two paths have "exchange" time orderings. However, all these four paths are not sensitive to the on-site Coulomb energy since no same-spin electrons can occupy the same energy level of the quantum dot. After summing all contributions from the virtual paths, we get the following transition amplitudes,

$$\begin{aligned}
\langle \uparrow, s | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= \frac{-2\sqrt{2}\Delta_L U^2 V_L^{*2} V_{R_1} V_{R_2}}{(E_L^2 - \Delta_L^2)(\Delta_L^2 - \Delta_R^2)((E_L - U)^2 - \Delta_L^2)} \\
\langle \uparrow, t | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= 0 \\
\langle \downarrow, \uparrow\uparrow | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= 0 \\
\langle \downarrow, \downarrow\downarrow | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= 0.
\end{aligned} \tag{5}$$

Obviously, only the spin-singlet final state can get nonzero amplitude after same-spin input

electrons co-tunneling through the dot to two output leads. Again, turning off of the on-site Coulomb energy will lead to vanishing spin-singlet amplitude. On the other hand, there is also no net amplitude for singlet final states if the two input same-spin electrons are of the same energy, i.e.,  $\Delta_L = 0$ . Similar to the case in Ref.[14], the increase of  $U$  will not lead to vanishing singlet amplitude. It simply suppresses the contributions from all the virtual co-tunneling paths but the paths (I, II) in Fig.3.

Next we consider the case that  $(\sigma\sigma') = (\downarrow\uparrow)$ . In this case, the initial state of the quantum dot system is  $\hat{c}_{(\downarrow)}^+ \hat{a}_{L,k,(\downarrow)}^+ \hat{a}_{L,k',(\uparrow)}^+ |0\rangle$ . The output electrons at the two right leads may form a spin-triplet, a spin-singlet, or a same-spin ( $\downarrow\downarrow$ ) state which corresponds to a spin-down, a spin-down, or a spin-up dot electron confined within the dot, respectively. Therefore, the possible final states of the dot system are  $|\downarrow, s\rangle, |\downarrow, t\rangle \equiv \frac{1}{\sqrt{2}} \hat{c}_{(\downarrow)}^+ (\hat{a}_{R_1(\uparrow)}^+ \hat{a}_{R_2(\downarrow)}^+ \mp \hat{a}_{R_1(\downarrow)}^+ \hat{a}_{R_2(\uparrow)}^+) |0\rangle$ , and  $|\uparrow, \downarrow\downarrow\rangle \equiv \hat{c}_{(\uparrow)}^+ \hat{a}_{R_1(\downarrow)}^+ \hat{a}_{R_2(\downarrow)}^+ |0\rangle$ . In this case, there are twelve virtual paths that contribute to the transition amplitudes for the spin-singlet or the spin-triplet final state. Six out of the twelve paths which correspond to "direct" time ordering are shown in Fig.4. The remaining six paths then correspond to "exchange" time ordering. On the other hand, there are eight virtual paths contributing to the transition amplitude for  $|\uparrow, \downarrow\downarrow\rangle$  final state. After summing all virtual paths, we obtain the following transition amplitudes:

$$\begin{aligned} \frac{\langle \downarrow, s | \hat{T}(\varepsilon_i) | \varphi_i \rangle}{V_L^* V_{R_1} V_{R_2}} &= \frac{\sqrt{2}U \{ (E_L U - E_L^2 + \Delta_R^2)(\Delta_L^2 - \Delta_R^2 + \Delta_L U) + U^2 \Delta_R^2 \}}{(E_L^2 - \Delta_R^2)(\Delta_L^2 - \Delta_R^2)((E_L - U)^2 - \Delta_R^2)(E_L + \Delta_L)(E_L - \Delta_L - U)} \\ \frac{\langle \downarrow, t | \hat{T}(\varepsilon_i) | \varphi_i \rangle}{V_L^* V_{R_1} V_{R_2}} &= \frac{-\sqrt{2}\Delta_R U^2}{(E_L^2 - \Delta_R^2)(\Delta_L^2 - \Delta_R^2)((E_L - U)^2 - \Delta_R^2)} \\ \langle \uparrow, \downarrow\downarrow | \hat{T}(\varepsilon_i) | \varphi_i \rangle &= -\langle \downarrow, t | \hat{T}(\varepsilon_i) | \varphi_i \rangle. \end{aligned} \quad (6)$$

The above result shows that net amplitudes may exist for all the final states  $|\downarrow, s\rangle, |\downarrow, t\rangle$  and  $|\uparrow, \downarrow\downarrow\rangle$  if  $U \neq 0$ . All amplitudes vanish when  $U = 0$ . However, there is a chance that the transition amplitude for  $|\downarrow, s\rangle$  final state will vanish:

$$(E_L U - E_L^2 + \Delta_R^2)(\Delta_L^2 - \Delta_R^2 + \Delta_L U) + U^2 \Delta_R^2 = 0. \quad (7)$$

For example, while  $E_L < 0$  and  $\Delta_L < \Delta_R$  are presumed in this paper, the above condition leads to  $\Delta_R^2 \simeq |E_L| \Delta_L$  in the  $U \rightarrow \infty$  limit. In this situation only the spin-triplet  $|\downarrow, t\rangle$  and the same-spin state  $|\uparrow, \downarrow\downarrow\rangle$  are allowed as the final states for the dot system.

Based on the above results, we find that the dot system may generate each of the four possible final states including  $|\downarrow, s\rangle, |\downarrow, t\rangle, |\uparrow, s\rangle$  and  $|\uparrow, \downarrow\downarrow\rangle$ . When the on-site



Coulomb energy is turned off, the transition amplitudes of all these final states also vanish. Otherwise, the last three final states always have non-zero transition amplitudes no matter what the model parameters are chosen. On the other hand, the transition amplitude for  $|\downarrow, s\rangle$  may be tuned to zero by carefully selecting the model parameters. This novel property guarantees that once the final dot electron is measured to be of the same spin as that of the initial dot electron, we can fully infer that a pair of entangled electrons which form a spin-triplet state are generated at the two output leads.

## V. CONCLUSION

In this paper, we discussed the possibility of using non-empty quantum dot as the electron entangler. The dot system consists of a single quantum dot which has only a single energy level within the dot, one input lead which is filled of electrons, and two output leads which are empty. Two situations are discussed in this paper : the dot system occupied by one, or by two dot electrons within the quantum dot. The dot system is arranged such that any single-electron tunneling process is suppressed due to energy conservation. Only two input electrons can co-tunnel through the dot to the output leads.

When the dot is occupied by two dot electrons, we found that the dot system can be a good electron entangler which always filters the single-state portion of the two-electron input and generates a non-local spin-singlet state at the output leads. This result is similar to that in Ref.[14] which considered an empty quantum as the electron entangler. In both systems, the on-site Coulomb interaction  $U$  acts as a nonlinear mediator which mediates electron entanglement. When  $U = 0$  all transition amplitudes vanish including the amplitude for the spin-singlet final state. When the on-site Coulomb energy is turned on, both systems generate only spin-singlet state at the output leads.

When the dot is occupied by a single dot electron with definite spin (assumed spin-down in the paper), we found that the dot system no longer filters any particular portion of the two-electron input. In general, there are nonzero transition amplitudes for four different final states:  $|\downarrow, s\rangle$ ,  $|\downarrow, t\rangle$ ,  $|\uparrow, s\rangle$ , and  $|\uparrow, \downarrow\rangle$ . Again, all these four amplitudes vanish when the Coulomb energy  $U$  is turned off. When  $U$  is turned on, the amplitudes for the last three final states are never vanished except for  $\Delta_L = 0$ . However, when Eq.(7) is satisfied the amplitude for the final state  $|\downarrow, s\rangle$  can be tuned to zero. If this is the case, only the

triplet final state  $|\downarrow, t\rangle$  will not flip the spin of its dot electron after co-tunneling. This means that when the dot electron is found to remain in the same spin, a pair of entangled electrons which form a spin-triplet must be generated at the output leads.

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- [1] D. Deutsch, Proc. R. Soc. Lond. A **400**, 97 (1985).; D. S. Simon, *Proceedings of the 34th Annual Symposium on the Foundations of Computer Science* (IEEE Press, Los Alamitos, 1994).
  - [2] P. W. Shor, *Proceedings of the 35th Annual Symposium on the Foundations of Computer Science* (IEEE Press, Los Alamitos, 1994), p. 124.; L. K. Grover, *Proceedings of the 28th Annual ACM Symposium on the Theory of Computing (STOC)*, p.212.;
  - [3] D. G. Cory, A. F. Fahmy and T. F. Havel, in *PhysComp96* (New England Complex Systems Institute, 1996), p.87.; J. A. Jones and M. Mosca, J. Chem. Phys. **109**, 1648 (1998).; I. L. Chuang, N. Gershenfeld and M. Kubinec, Phys. Rev. Lett. **80**, 3408 (1998).; N. Linden, H. Barjat and R. Freeman, Chem. Phys. Lett **296**, 61 (1998).
  - [4] H.-J. Briegel, T. Calarco, D. Jaksch, J. I. Cirac and P. Zoller, in *Quantum Computation and Quantum Information Theory* (World Scientific Publishing Co.,Singapore, 2000).
  - [5] Y. Makhlin, G. Schon and A. Shnirman, Nature **398**, 305 (1999).;A. Shnirman, G. Schon and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
  - [6] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).; H. C. Nagerl, W. Bechter, J. Eschner, F. Schmidt-Kaler, R. Blatt, Appl. Phys. B **66**, 603 (1998).; D. F. V. James, Appl. Phys. B **66**, 181 (1998).
  - [7] H. Mabuchi and P. Zoller, Phys. Rev. Lett. **76**, 3108 (1996).;C. J. Hood, M. S. Chapman, T. W. Lynn and H. J. Kimble, Phys. Rev. Lett. **80**, 4157 (1998).;X. Maitre, E. Hagley, G. Nogues, C. Wunderlich, P. Goy, M. Brune, J. Raimond and S. Haroche, Phys. Rev. Lett. **79**, 769 (1997).

- [8] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); G. Burkard and D. Loss, Phys. Rev. B **59**, 2070 (1999).
- [9] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935).
- [10] C. H. Bennett, D. DiVincenzo, J. A. Smolin and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
- [11] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
- [12] C. H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India* (IEEE, New York, 1984), p. 175.; A. K. Ekert, Phys. Rev. Lett, **67**, 661 (1991).; C. H. Bennett, Phys. Rev. Lett, **68**, 3121 (1992).; P. D. Townsend, Nature **385**, 47 (1997).; E. Biham, B. Huttner, and T. Mor, Phys. Rev. A **54**, 2651 (1996).;
- [13] A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).; G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *ibid.* **81**, 5039 (1998).
- [14] W. D. Oliver, F. Yamaguchi and Y. Yamamoto, Phys. Rev. Lett. **88**, 037901 (2002).
- [15] P. Recher, E. V. Sukhorukov and D. Loss, Phys. Rev. B **63**, 165314 (2001).; J. Schliemann, D. Loss and A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001).
- [16] A. Bertoni, P. Bordone, R. Brunetti, C. Jacoboni and S. Reggiani, Phys. Rev. Lett. **84**, 5912 (2000).
- [17] D. Loss and E. V. Sukhorukov, Phys. Rev. Lett. **84**, 1035 (2000).; G. Burkard, D. Loss and E. V. Sukhorukov, Phys. Rev. B **61**, R16303 (2000).
- [18] J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).

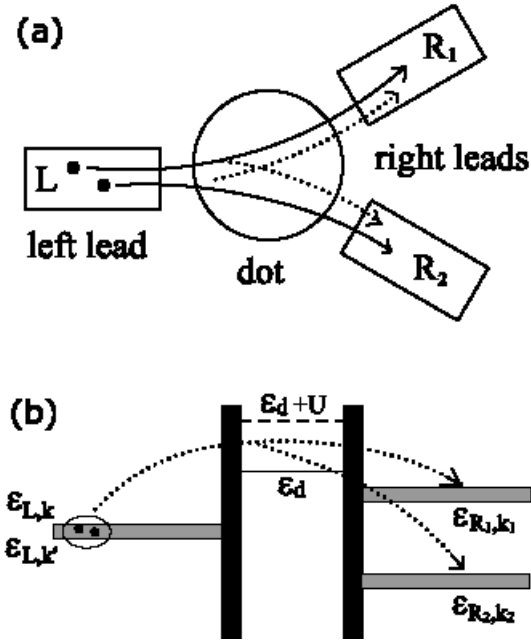


FIG. 1: (a) The quantum dot system with three leads. (b) The energy band diagram of the quantum dot system.

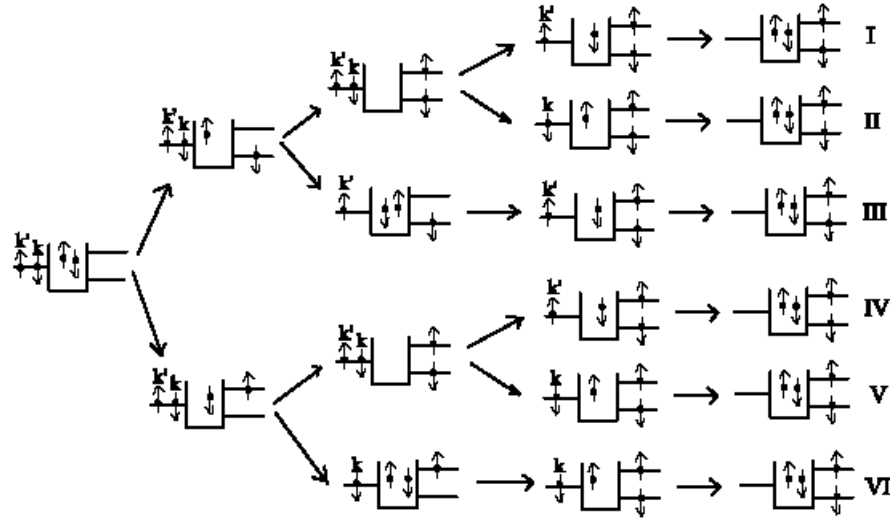


FIG. 2: Multiple paths by which two input electrons of different spins virtually co-tunnel through a quantum dot occupied by two electrons to two output leads.

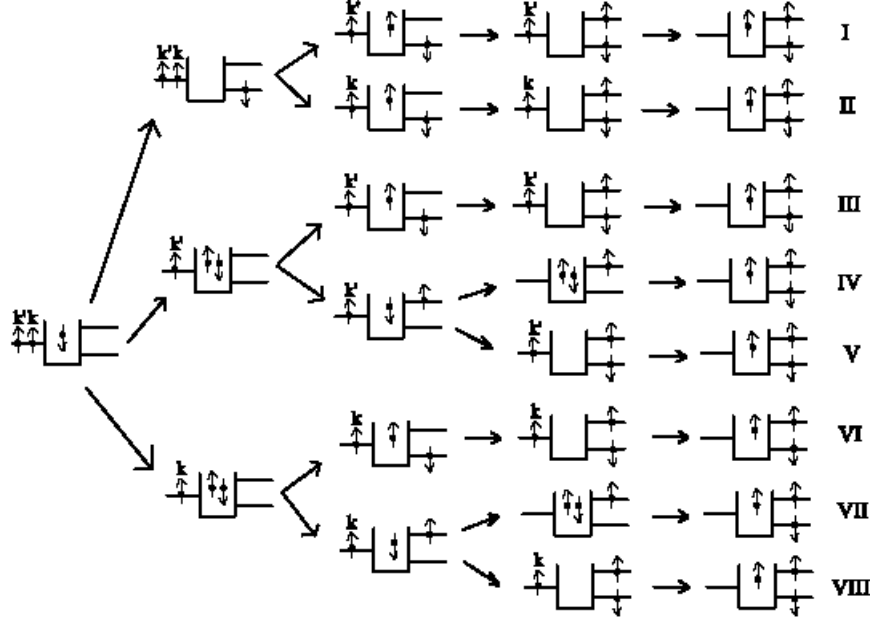


FIG. 3: Multiple paths by which two same-spin electrons virtually co-tunnel through a quantum dot occupied by a single electron to two output leads.

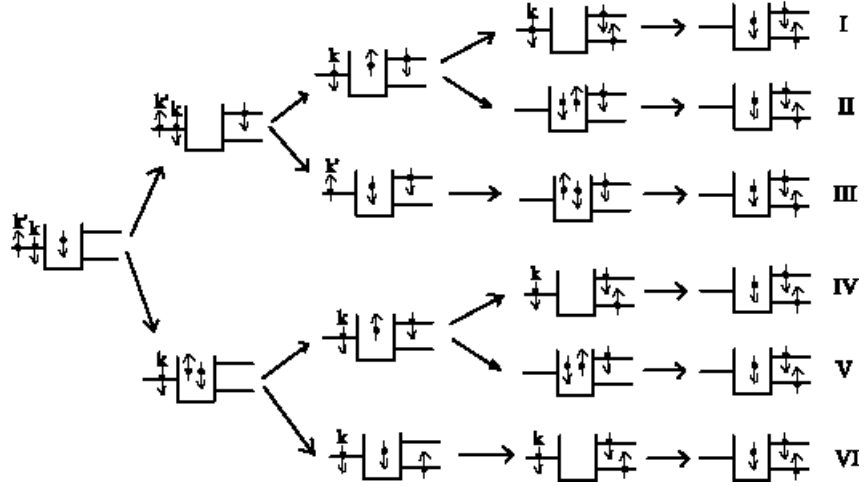


FIG. 4: Multiple paths by which two different-spin electrons virtually co-tunnel through a quantum dot occupied by a single electron to two output leads.